Bowdoin College

Math 3603: Advanced Analysis
Prof. Thomas Pietraho
Spring, 2022

Syllabus

Course Description: Measure theory and integration with applications to probability and mathematical finance. Topics include Lebesgue measure and integral, measurable functions and random variables, convergence theorems, analysis of random processes including random walks and Brownian motion, and the Itô integral

Objectives: At the end of the course, a student should be able to:

1. define and be able to work with the concepts of measure and the Lebesgue integral;
2. use measure theory to address and answer questions in probability;
3. understand models for random motion including random walks and Brownian motion;
4. define and work with the Itô stochastic integral;
5. appreciate and critique the applications of the above topics in mathematical finance.

Web Page: The class web page can be accessed from:

https://pietraho.com

I will use this page to post homework, tests, as well as their solutions. It also contains a copy of this handout.

Meeting Times: The class meets Mondays and Wednesdays at 2:50PM in Searles 213.

Contacting Me: Here is a list of my coordinates:

Office:	Searles 205
Phone:	725-3568
Email:	tpietrah@bowdoin.edu

The sure-fire method of contacting me is by email.

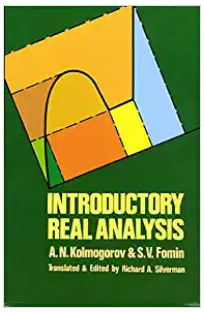
Office Hours: I will hold official weekly office hours during the following times:

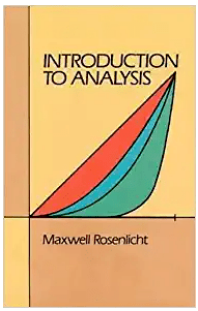
Monday	4:15PM- 5:00PM
Tuesdayday	4:15PM- 5:00PM
Wednesday	4:15PM- 5:00PM

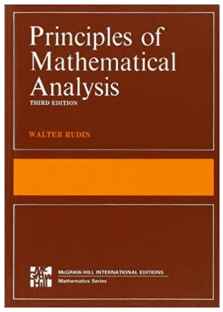
Alternately, feel free to make an appointment at a different time, or just drop by my office; if I am in, I will be happy to see you. In addition, special office hours will be scheduled before exams and the final.

REFERENCES: There will be no required course textbook. The following is a list of references you may find useful during the different parts of the course. A few non-circulating copies of these books will be available in the math department library in Searles 214.

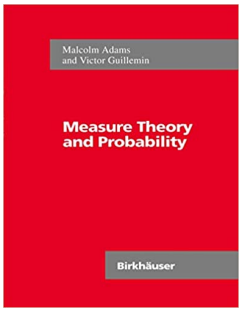
For elementary real analysis:

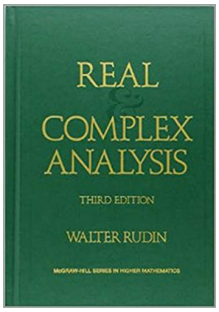


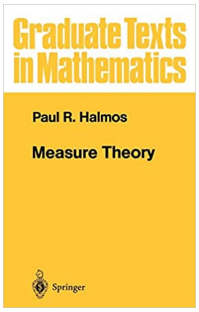




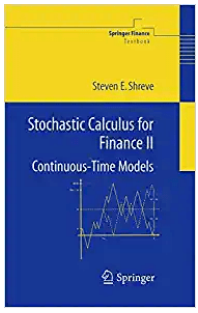
For measure theory and integration:







For financial mathematics:



YOUR GRADE: Your grade will be based on homework and exam grades.

- Homework will be assigned at every class meeting and will be due at 9pm the following Tuesday unless otherwise indicated. You are permitted, and in fact, encouraged, to work with others on the homework. However, you must write up the solutions independently and cite your collaborators.
- There will be two exams during the course of the semester, both take-home assignments. The first will be due before our spring break, and the second will be due during finals week. Precise details will be announced in class.

Your final grade will be based on the total number of points earned in the class on the homework and exams.

STUDY HYGENE: This is an advanced course and while it will demand more time than Math 2603, this increase will be quite modest. What will be different is how you will spend your time. As content gets more sophisticated, it will become more and more important to spend time reviewing your notes carefully, especially before each class so you are ready to engage in the conversation from the start. Recognizing this, homework assignments will be balanced: the shorter the homework, the more time I expect you to spend digesting the material in your notes.

READING AND WATCHING MATHEMATICS: Learning mathematics is not a spectator sport. Reading mathematics is not like reading a novel; watching mathematics is not like watching an action thriller. Some paragraphs are easy to digest, but you may find yourself looking at one line of text for five or more minutes trying to understand what the author is trying to say. Use the pause button when watching a video. As you read or watch, take notes, just as you do in class. This is crucial! If questions arise, write them down and ask during office hours.

TOPICS COVERED: The following is a tentative outline of the course. I reserve the right to introduce additional topics as student and intructor interest permit.

1. Introduction
2. Measure Theory
   (a) Introduction
      i. Measures on Rings of Sets
      ii. Outer Measure
      iii. Lebesgue Measure
   (b) Measure Theory and Probability
      i. Probability Measure
      ii. Bernoulli Sequences and Random Walks
      iii. "Infinite Monkey" Theorem
   (c) Measure Theory and Integration
      i. Measurable Functions and Random Variables
      ii. Integration of Simple Functions
      iii. Lebesgue Integral
      iv. Tchebyshev's Theorem and Convergence Theorems
3. Random Variables

(a) Random Variables and Probability Distributions
(b) Expected Value, Variance, and Higher Moments
(c) Independence
(d) Law of Large Numbers
(e) Central Limit Theorem and Applications

4. Stochastic Processes
   (a) Discrete Brownian Motion
   (b) Continuous Brownian Motion
      i. Existence of Continuous Brownian Motion
      ii. Properties of Continuous Brownian Motion
      iii. Variation and Quadratic Variation
   (c) Gaussian Processes

5. Stochastic Calculus
   (a) Motivation
   (b) Itô Integral and General Stochastic Integrals
   (c) Itô Processes and the Itô Formula

6. Finance and the Black-Scholes-Merton Formula
   (a) Introduction to Options and Financial Derivatives
   (b) A Portfolio Hedging Model
   (c) Derivation of the Black-Scholes-Merton Formula
   (d) Applications and Nassim Taleb's Critique