Bowdoin College

Homework 6

1. Consider the following definition from class:

   Definition. *The characteristic, or indicator, function χ_A of a set A is defined by*

   $$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \text{ and} \\ 0 & \text{if } x \notin A \end{cases}$$

   The purpose of this exercise is to define a relationship between the definitions of $\liminf$ and $\limsup$ on sequences of sets versus sequences of real numbers. Consider a set X and a collection of subsets $\{A_i\}_{i=1}^{\infty}$. Write

   $$A_+ = \limsup A_i \qquad \text{and} \qquad A_- = \liminf A_i$$

   Let $f_+ = \chi_{A_+}$, $f_- = \chi_{A_-}$, and $f_n = \chi_{A_n}$, be the characteristic functions of the corresponding sets. Prove that $f_+ = \limsup f_n$ and $f_- = \liminf f_n$

2. Suppose that f is a non-negative measurable function on X and $A, B \subset X$ are disjoint measurable sets. Show that

   $$\int_{A \sqcup B} f \, d\mu = \int_A f \, d\mu + \int_B f \, d\mu.$$