Bowdoin College

Homework 11

1. Consider a function $f : [a, b] \to \mathbb{R}$ which may or may not be Riemann integrable. For any two partitions P and Q of the interval $[a, b]$, show that

$$L(f, P) \leq U(f, Q),$$

   that is, *any* upper sum is greater than or equal to *any* lower sum for f.

   Hint: Use a common refinement, that is, a partition that is both a refinement of P and Q.

2. Consider two Riemann-integrable functions $f, g : [a, b] \to \mathbb{R}$. Show that if for all $x \in [a, b]$ we have $f(x) \leq g(x)$, then

$$\int_a^b f \leq \int_a^b g.$$