Monday, August 3, 2020 12:59 PM

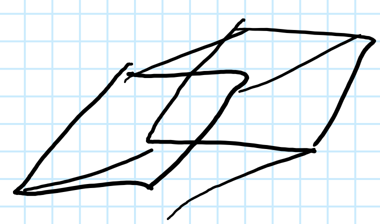
DETERMINANT:

MANY DEFINITIONS.

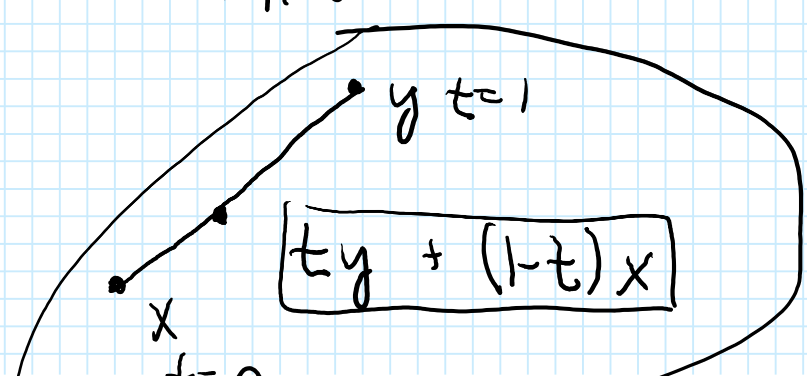
GEO. OBSERVATION

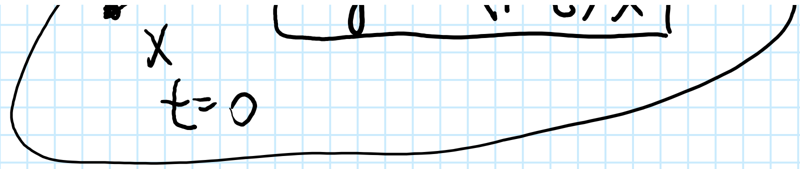
$$A : \mathbb{R}^n \to \mathbb{R}^n$$

FACT: THE IMAGE OF A CUBE UNDER A IS A PARALLELOPIPED.



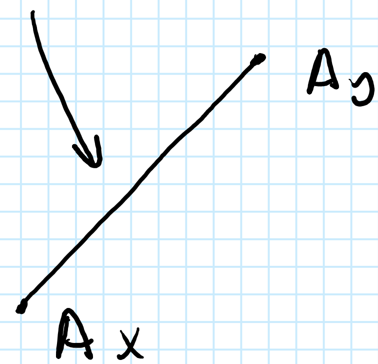
PF: LINE SEGMENTS GO TO LINE SEGMENTS.





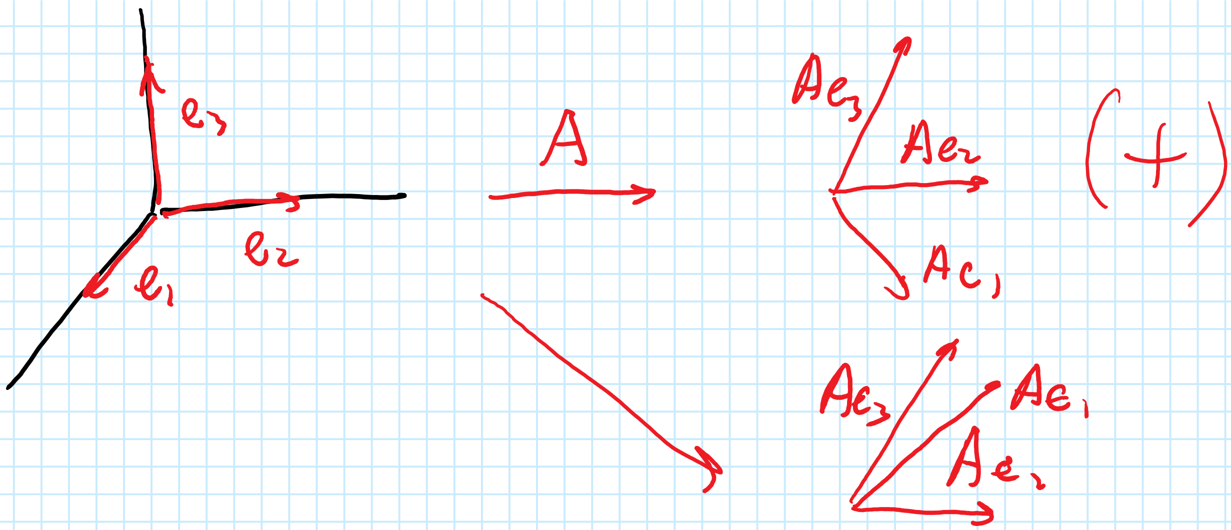
$$A\big(ty + (1-t)x\big) = A(ty) + A\big((1-t)x\big)$$

$$= \underline{t(Ay) + (1-t)Ax}$$



DEF: $\det A =$ (SIGNED) VOLUME OF THE PARALLELOGRAM IF A IS APPLIED TO THE UNIT CUBE.

$$\text{SIGN} = \begin{cases} + & \text{IF SAME ORIENTATION} \\ & \text{IF DIFF ORIENTATION.} \end{cases}$$



FACT: ALL FORMULAS FOR $\det A$ CAN BE DERIVED FROM THIS.

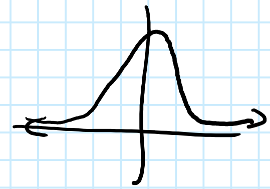
FACT: A IS INVERTIBLE (A BIJECTION) IFF $\det A \neq 0$.

INTUITION: VOLUME GOES TO 0. CAN'T BE AN INJECTION.

RANDOM MATRIX. RAND (UNIFORM).

$$A = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

RANDOM (NORMAL).



QUESTION: WHAT IS $\det A$ FOR A RANDOM MATRIX?

DOES IT DEPEND ON HOW WE PICK RANDOM #s?

EX: COVARIANCE MATRIX.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \\ & & a_{ij} & \\ & & & \end{pmatrix}$$

COV. OF i WRT j