Monday, July 6, 2020 12:56 PM

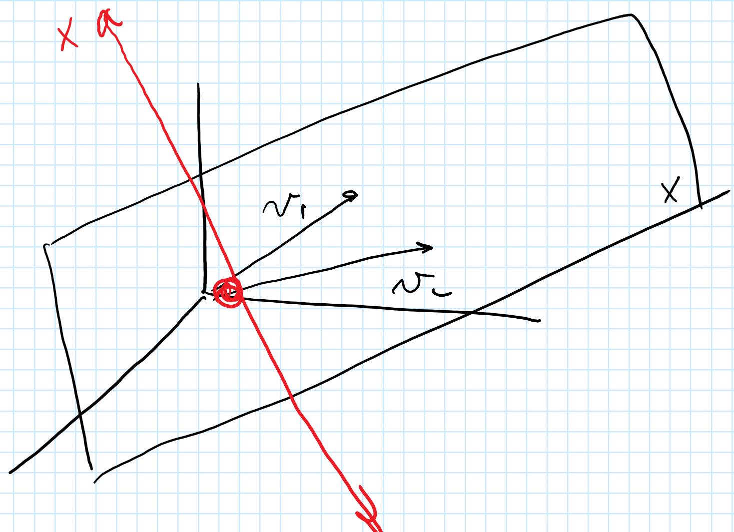
EXAM: UPCOMING. WILL ANNOUNCE.

LAST TIME: SOLUTIONS TO $Ax = \vec{0}$

(NULLSPACE).

THIS IS A VECTOR SPACE.

PICTURE:



SOMETHING LIKE:

$$x_1 \underbrace{\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}}_{v_1} + x_2 \underbrace{\begin{pmatrix}0\\0\\1\\-1\end{pmatrix}}_{v_2}$$

NEXT: $Ax = b$

IDEA: FIND A SINGLE x_p SO THAT

$$Ax_p = b$$

PARTICULAR SOLN.

OTHER SOLUTIONS?

SUPPOSE $Ax_n = 0$

SUPPOSE $A x_n = 0$

IN NULLSPACE.

THEN COMBINE:

$$A(x_p + x_n) = Ax_p + Ax_n$$
$$= b + 0 = b$$

NOTE: IF x_n IS IN NULLSPACE FOR A,

& x_p IS A PARTICULAR SOL'N,

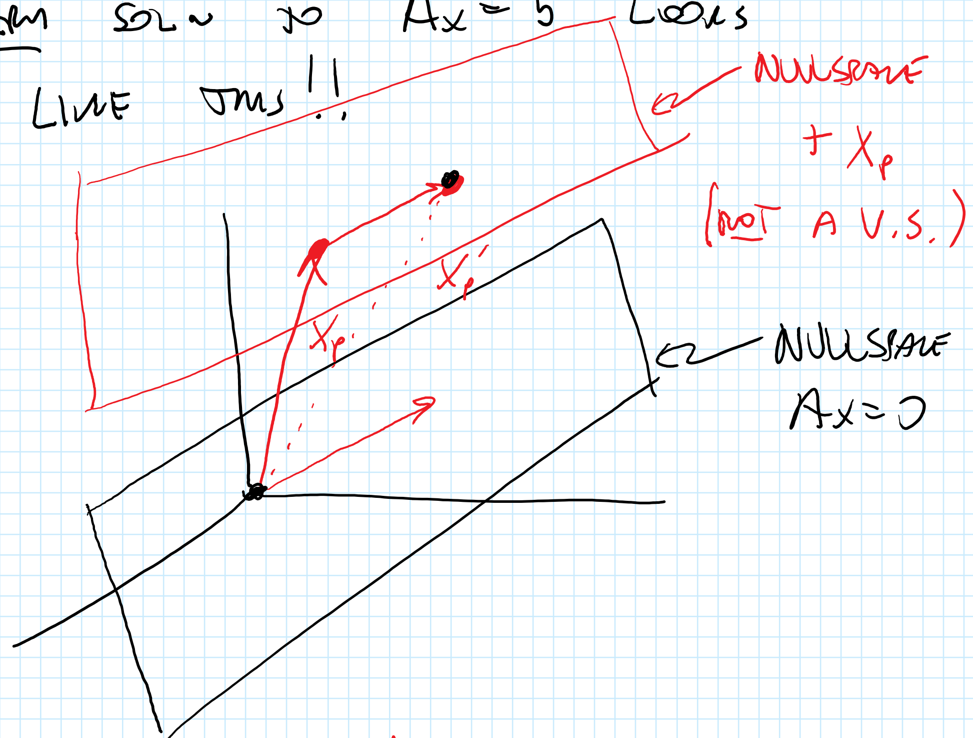
THEN EVERY

$$x_p + x_n$$

IS A SOL'N TO $Ax = b$!

THEN: EVERY SOL'N TO $Ax = b$ LOOKS LIKE THIS!!

PICTURE:

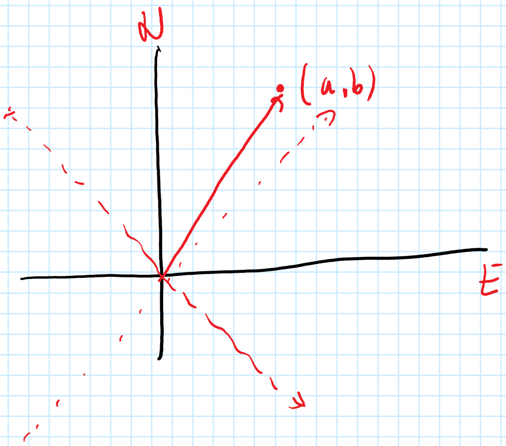


$Ax = b$ $\quad Ay = b$ $\quad Ax - Ay = 0 = A(x-y) = 0$

FORESHADOWING: WILL SEE THIS IN O.D.E.

BASIS:

IDEA: FIND A SIMPLE REP'N OF A VECTOR.



$$\boxed{a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}}$$

ALSO:

$$\boxed{a'\begin{pmatrix}1\\1\end{pmatrix} + b'\begin{pmatrix}-1\\1\end{pmatrix}}$$

BASIS:

- EVERY VECTOR IN OUR VECTOR SPACE IS A LIN. COMB. OF "BASIS" VECTORS.
- THERE ARE NO UNNECESSARY VECTORS.

$$\begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} , \cancel{\begin{pmatrix}1\\1\end{pmatrix}}$$

QUESTIONS

- UNNECESSARY?
- HOW DO YOU KNOW YOU HAVE A BASIS?

OTHER SCENARIOS:

(1) FIND A BASIS FOR ALL FUNCTIONS

$f : \mathbb{R} \to \mathbb{R}$

TAYLOR SERIES:

$$e^x = 1 + 1 \cdot x + \frac{x^2}{2} + \frac{x^3}{6} + , \dots$$

$$x^0 \quad x \quad \frac{1}{2} \cdot x^2 \quad \frac{1}{6} x^3 \quad \dots$$

Q: $\{1, x, x^2, x^3, \dots\}$

IS THIS A BASIS FOR FUNCTIONS?

<u>EX</u>:

$$e^{-\frac{1}{x^2}} \neq 0 + 0x + 0x^2 + 0x^3 + 0x^4 + , \dots$$