Monday, June 29, 2020 12:53 PM

(1) POETRY VS MATH:

POINCARÉ:

- POETRY IS ART OF GIVING MANY DIFF NAMES TO SAME THING.
- MATHEMATICS IS ART OF GIVING THE SAME NAME TO MANY DIFF THINGS.

VECTOR SPACES:

(1) VECTORS IN $\mathbb{R}^n$
(2) POLYNOMIALS
(3) FUNCTIONS
(4) MATRICES.

COMMONALITIES:

- CAN ADD TWO "THINGS"
- CAN MULTIPLY ON A SCALAR.

SIMILAR PROPERTIES:

- $\alpha(v+w) = \alpha v + \alpha w$

SAME IS TRUE FOR ALL OF ABOVE!

- $(\alpha+\beta)v = \alpha v + \beta v$

IDEA: GIVE (1)-(4) A COMMON NAME:
VECTOR SPACES.
PROVE THINGS ABOUT VECTORSPACES
IN GENERAL.

(2) MATRICES AS FUNCTIONS.

EX:
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

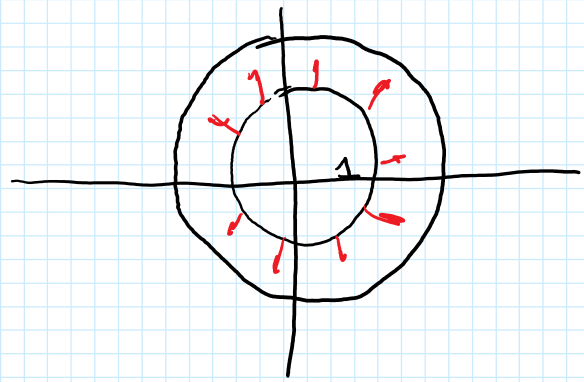
A

IDEA: A IS A FUNCTION!

$$A: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

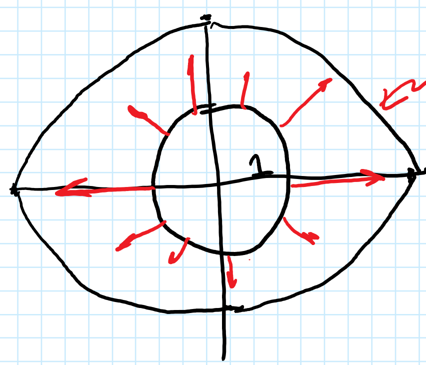
QUESTION: HOW TO VISUALIZE
SUCH A FUNCTION?

EX:
$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$$



EX:

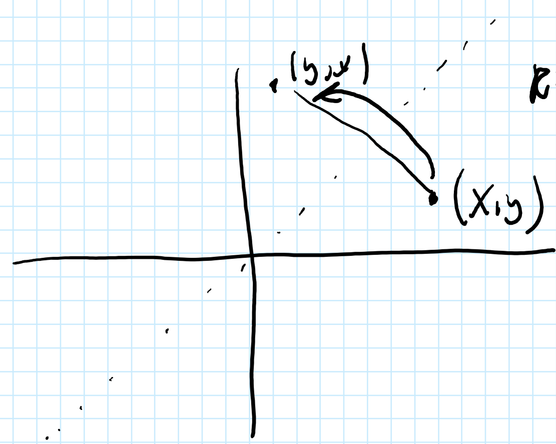
$$\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}3x\\ 2y\end{pmatrix}$$



$(3x)^2+(2y)^2=1$?

EX:

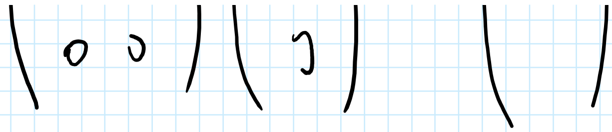
$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}y\\ x\end{pmatrix}$$

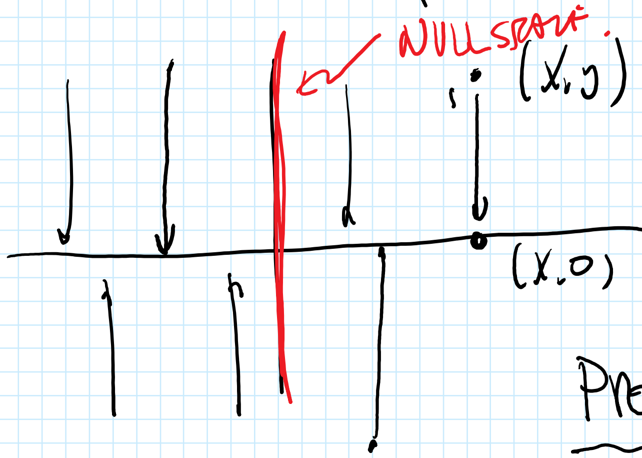


REFLECTION!

EX:

$$\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ 0\end{pmatrix}$$





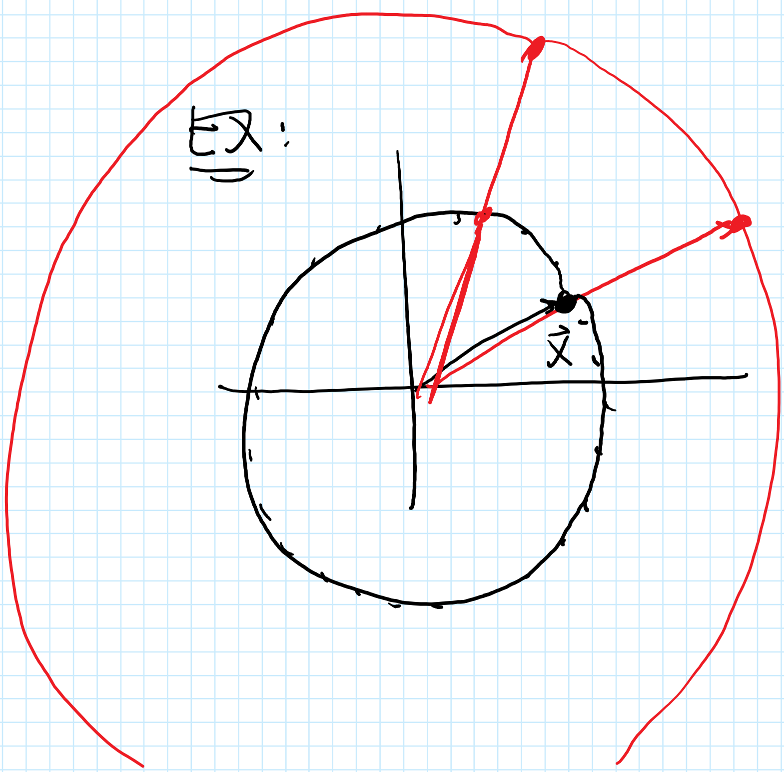
PROJECTION

GOAL: BY END OF CLASS

VISUALIZE A MATRIX

AS A FUNCTION....

EX:



$A\vec{x} = \vec{b}$

IDEA: AB ~ A FUNCTION.

IT IS $A(Bx)$

AS FUNCTIONS,

$A \cdot B$ IS THE FUNCTION OBTAINED BY COMPOSING THE TWO FUNCTIONS $A \circ B$.

QUESTION:

$$f\underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \underbrace{\begin{pmatrix} a & c \\ b & d \end{pmatrix}}$$

IS f A MATRIX ??

CAN YOU FIND A MATRIX

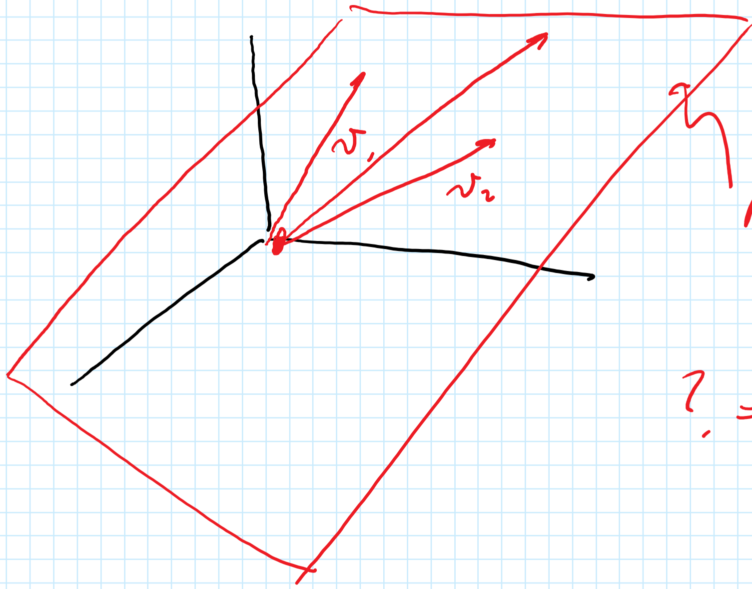
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

THAT WORKS? (**)

(1) ROWSPACE OF A MATRIX:

$$A = \begin{pmatrix} - & v_1 & - \\ - & v_2 & - \\ - & v_3 & - \end{pmatrix}$$

$$M = \begin{pmatrix} - & v_2 & - \\ - & v_3 & - \end{pmatrix}$$



ALL COMBINATIONS OF v_1 v_2 v_3

$? = a v_1 + b v_2 + c v_3$

(2) NULLSPACE

$$A\vec{x} = \vec{0}$$

WHICH VECTORS GO TO $\vec{0}$.