Monday, June 22, 2020 12:32 PM

NOTES

HISTORY: GAUSS-JORDAN ELIMINATION
DATES BACK TO NEWTON
IF NOT ANCIENT CHINA.

INVERSES OF MATRICES

$$A(Bv) = (AB)v$$

SUPPOSE YOU WANT TO UNDO THIS:

$$B^{-1}A^{-1}ABv = B^{-1}(A^{-1}A)Bv$$

$$= B^{-1}IBv = B^{-1}Bv = Iv = v$$

THUS: $(AB)^{-1} = B^{-1}A^{-1}$

THINK SOCKS & SHOES.

ELIMINATION AS MATRIX MULT.

$$\boxed{E_k \cdots E_2 E_1 A = U}$$

$$\boxed{E_k \cdots E_2 E_1 A = U}$$

NOTE: EACH ELIMINATION MATRIX IS LOWER TRIANGULAR. (L.T.)

THRM: (1) IF E_1 & E_2 ARE L.T. THEN SO IS $E_1 \cdot E_2$.

$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} \begin{pmatrix} a' & 0 & 0 \\ b' & c' & 0 \\ d' & e' & f' \end{pmatrix}$$

$$= \begin{pmatrix} \cdot & 0 & 0 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot \end{pmatrix}$$

(2) IF E IS L.T., THEN SO IS E^{-1}.

IN SUMMARY:

ELIMINATION:

$$MA = U$$

BY THRM (1), THIS IS L.T. IT IS $E_k \cdots E_2 E_1$ AS ABOVE.

THUS

$$A = M^{-1} U$$

$$= LU$$

ALSO L.T. BY (2).

THIS ALMOST ALWAYS WORKS. SOMETIMES, NEED TO PERMUTE FIRST:

$$\boxed{PA = LU}$$

PERMUTATION MATRIX.

NOTES:

- $A = LU$ IS A WAY OF FACTORING A MATRIX INTO SIMPLE PARTS.
- SYMMETRIC MATRICES.
- MATRICES ENCODE "SYMMETRY".