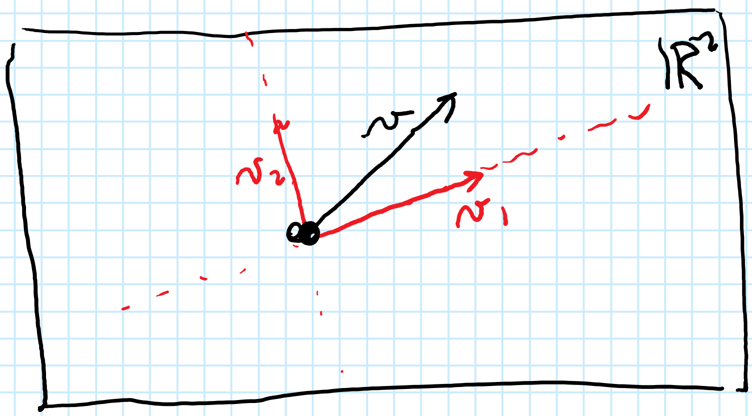
DIAGONALIZATION:



(1) TAKE $v \in \mathbb{R}^n$. NO BASIS YET.

(2) CHOOSE A BASIS $\mathcal{B} = \{v_1, v_2\}$

NOT NEC. ORTHOGONAL.

(3) WRITE v AS A LIN. COMB OF v_1 & v_2.

$$v = \alpha_1 v_1 + \alpha_2 v_2$$

NOTE: · CLAIM: CAN ALWAYS DO THIS. SINCE $v \in \text{SPAN}\{v_1, v_2\}$

· EXPRESSION IS UNIQUE. v_1 & v_2 ARE INDEP.

PF:

$$v = \alpha_1 v_1 + \alpha_2 v_2$$

$$v = \beta_1 v_1 + \beta_2 v_2$$

$$0 = (\alpha_1 - \beta_1)v_1 + (\alpha_2 - \beta_2)v_2$$

$$\boxed{0 = \gamma_1 v_1 + \gamma_2 v_2}$$

SINCE L.I., $\gamma_1 = \gamma_2 = 0$

I.E. $\alpha_1 = \beta_1$ & $\alpha_2 = \beta_2$ ⊠

(4) DEF: α_1 & α_2 ARE THE COORDINATES OF v WRT $\mathcal{B}$.

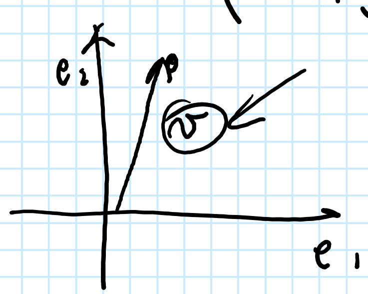
WRITE: $\boxed{v = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}_{\mathcal{B}}}$

NOTE: WE DO THIS ALL THE TIME.

BUT WITH $\mathcal{B} = S = \{e_1, e_2\}$

THEN, SUPPRESS NOTATION & WRITE

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}_S = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1e_1 + 2e_2.$$



QUESTION:

IF WE KNOW

$$v = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}_{B_1} \quad \& \quad v = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}_{B_2}$$

HOW TO CHANGE COORDINATES OF v FROM ONE BASIS TO ANOTHER?

FACT: THERE IS A MATRIX M SUCH THAT

$$M \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

IT IS THE CHANGE OF BASIS MATRIX.

OBSERVATION: WHEN WE WRITE A MATRIX

$$A = (a_{ij})$$

IT DEFINES A FUNCTION $A : \mathbb{R}^n \to \mathbb{R}^n$

GIVEN $v \in \mathbb{R}^n$ WRITTEN IN TERMS OF STANDARD BASIS IT TELLS US HOW TO FIND $Av \in \mathbb{R}^n$, IN TERMS OF STD. BASIS.

QUESTION: IF WE USE A DIFF BASIS

QUESTION: IF WE USE A DIFF BASIS FOR $\mathbb{R}^n$, HOW DO WE CHANGE THE MATRIX A APPROPRIATELY?!

ANS: $$A_B = M^{-1} A_S M$$

CHANGE OF BASIS MATRIX.

§ 6.2 SUPPOSE THE E. VECTORS OF A FORM A BASIS (NOT ALWAYS TRUE). WRITE A IN TERMS OF THIS BASIS

THE ANSWER WILL AMAZE YOU.

NOT ALWAYS POSSIBLE.

§6.4 EX.: THRM: IF A IS SYMMETRIC THEN:

(1) ALL E. VALUES ARE IN $\mathbb{R}$.

(2) E. VECTORS FORM A BASIS &

THEY ARE ORTHOGONAL!

EX: (1) SOCIAL NETWORKS

(2) HESSIAN

$$\left(\frac{\partial^2 f}{\partial x_i \, \partial x_j} \right)$$

$\boxtimes$

$$Av_1 = \lambda_1 v_1$$
$$Av_2 = \lambda_2 v_2$$

$$\boxed{\lambda_1 = \lambda_2}$$

$$Av = A(\alpha_1 v_1 + \alpha_2 v_2)$$

$$Av = \boxed{\alpha_1 \lambda_1 v_1 + \alpha_2 \lambda_2 v_2}$$

$$\boxed{A^k v = \boxed{\alpha_1 \lambda_1^k v_1 + \alpha_2 \lambda_2^k v_2}}$$