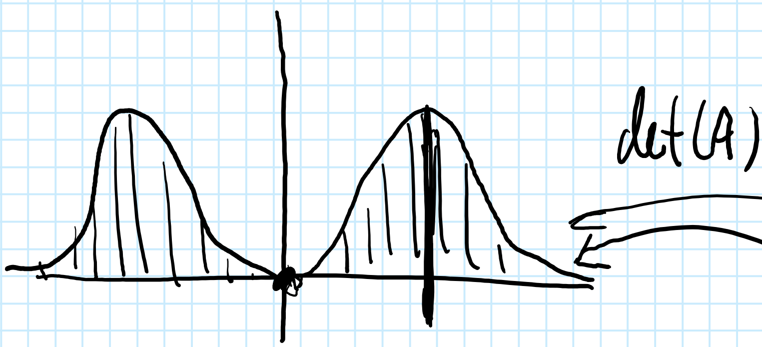
Monday, August 10, 2020 12:41 PM

RANDOM MATRICES

WHAT IS DET(A)

↳ RANDOM MATRIX?

IDEA: SAMPLE & SEE



f(x)

~~IDEA~~ Q: WHAT IS THIS CURVE?

(2010) T. TAO.

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

THIS WEEK:

EIGENVALUES & E.VECTORS. OF A

$$Ax = y$$

NOTHING TO DO WITH x.

SOMETIMES

SOME REAL #.

$$\boxed{Ax = \lambda x}$$

E.VALUE OF A

E.VECTOR OF A

SUPPOSE A IS A 2×2 MATRIX.

& HAS 2 E.VECTORS

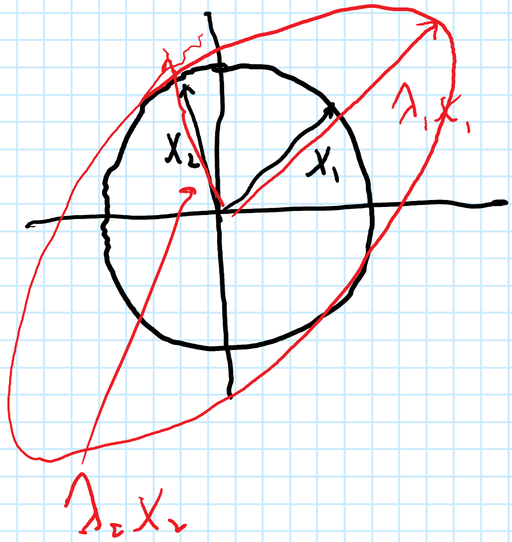
x_1 x_2

WITH E. VALUES

λ_1 λ_2

$$Ax_1 = \lambda_1 x_1 \qquad Ax_2 = \lambda_2 x_2$$

PICTURE:



Q: GIVEN A, HOW MANY E. VECTORS & E. VALUES DOES IT HAVE. FIND THEM!!

DYNAMICAL SYSTEMS

x_0 = INITIAL STATE OF A SYSTEM.

x_1 = STATE AT $t=1$

$\vdots$

x_n

OFTEN: $A x_{n-1} = x_n$

$$A X_{n-1} = X_n$$

$$A^n X_0 = X_n$$

(1) ~~[illegible]~~ WHAT HAPPENS IF n IS LARGE?

(2) IF $n \longrightarrow \infty$?

SOLUTION: · SUPPOSE A IS A $k \times k$ MATRIX &

$Z_1 \quad Z_2 \quad \ldots \quad Z_k$

ARE A SET E. VECTORS FOR A. WITH E VALUES $\lambda_1 \ldots \lambda_k$.

· SUPPOSE $Z_1 \ldots Z_k$ FORM A BASIS.

LET $\quad X_0 = c_1 z_1 + c_2 z_1 + \ldots + c_k z_k$

$$A x_0 = A(c_1 z_1 + \ldots + c_k z_k)$$

$$= A c_1 z_1 + \ldots + A c_k z_k$$

$$= c_1 A z_1 + \dots + c_k A z_k$$

$$= c_1 \lambda_1 z_1 + \dots + c_k \lambda_k z_k$$

$$A(Ax_0) = A(c_1 \lambda_1 z_1 + \dots + c_k \lambda_k z_k)$$

$$= c_1 \lambda_1 A z_1 + \dots + c_k \lambda_k A z_k$$

$$= c_1 \lambda_1^2 z_1 + \dots + c_k \lambda_k^2 z_k$$

THUS

$$x_n = \boxed{A^n x_0} = \cancel{c_1 \lambda_1^n z_1 + \dots} + c_k \lambda_k^n z_k$$

$$\approx \boxed{c_k \lambda_k^n z_k}$$

MIGRATION PATTERNS

$$A \qquad \begin{pmatrix} x_1 \\ \vdots \\ x_k \end{pmatrix}$$

POP. OF CITY k AT TIME $t=0$.

ENCODES HOW PEOPLE MOVE
FROM CITY TO CITY.