

Linear Algebra Course Themes

Thom Pietraho

Bowdoin College



Linear Algebra

Initially: Linear algebra introduced as a convenient way to solve systems of equations.

Three sheaves of good crop, 2 sheaves of mediocre crop, and 1 sheaf of bad crop are sold for 39 dou. Two sheaves of good, 3 of mediocre, and 1 of bad are sold for 34 dou. One sheaf of good, 2 of mediocre, and 3 of bad are sold for 26 dou. What is the price for a sheaf of good crop, mediocre crop, and bad crop?

-Chiu-chang Suan-shu, Nine Chapters in Arithmetic (200 BC)



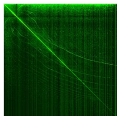
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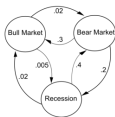
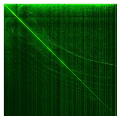
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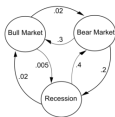
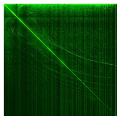
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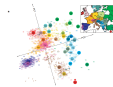
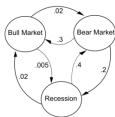
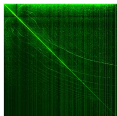
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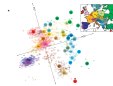
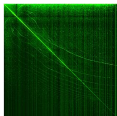
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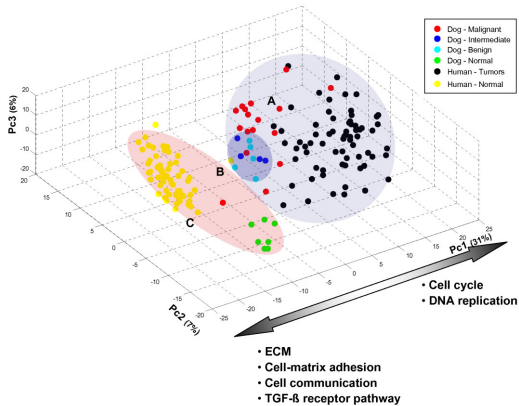
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Course Goal: Introduce the mathematical tools and techniques necessary to be able to use linear algebra in these contexts.



Application: Clustering



Tissue samples, Uva et al. (dark blue = benign, red = normal.)



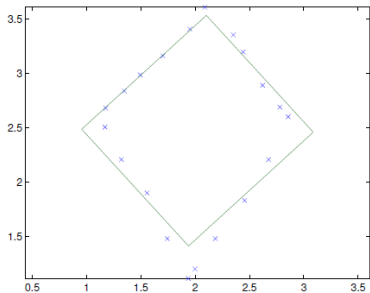
Application: Clustering



Voting records in Quebec.



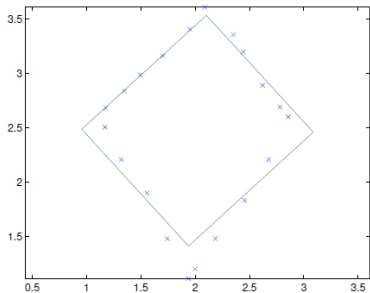
Application: Pattern Fitting



Fitting a particular shape to a set of sample points.



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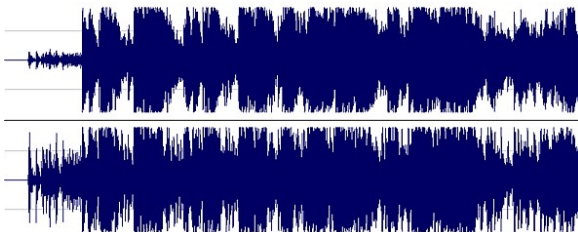


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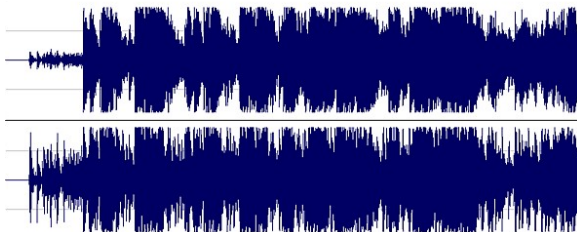
Example: Linear regression in statistics. Many more.



Application: Sound Compression



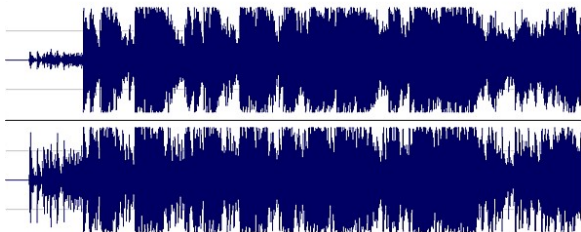
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This sounds very similar to the question above, but the theory is much different.



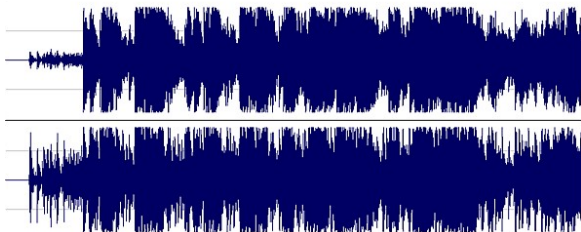
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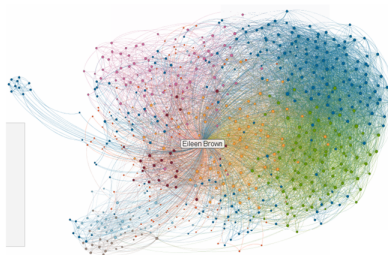
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Moral: As the class goes on, some of our results will be done in more general setting than \mathbb{R}^n .



Networks

Eileen Brown's Social Network:

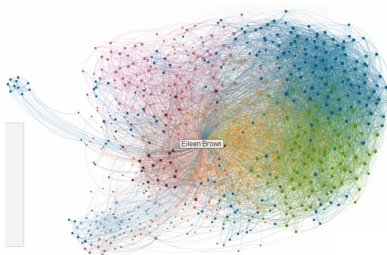


This can be encoded by a matrix.



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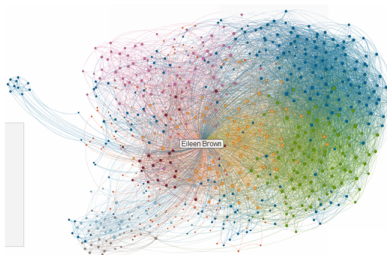
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Upshot: Can use linear algebra to analyze the graph.



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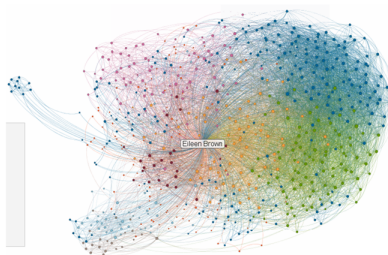
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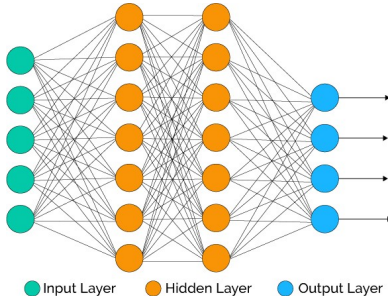
Questions: Find cliques, model disease propagation.

Question: How to graphically render the graph to display most information?



Neural Networks and Machine Learning

An Artificial Neural Network:



Machine learning employs a considerable amount of linear algebra, and it turns out that sometimes linear algebra can be *learned* by neural networks. But more on that later . . .

Image: [medium.com](https://www.medium.com)

