DCS/CSCI 2350: Social & Economic Networks

Matching Markets
Readings: Ch. 10 of EK & Handout for stable marriage

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60 Lives, 30 Kidneys, All Linked

RIVERSIDE, Calif. — Rick Ruzzonetti admits to being a tad impulsive. He traded his Catholicism for Buddhism in a revelatory flash. He married a Vietnamese woman he had only just met. And then a year ago, he decided in an instant to donate his left kidney to a stranger.

In February 2011, the desk clerk at Mr. Ruzzonetti's yoga studio told him she had recently donated a kidney to an ailing friend she had bumped into at Target. Mr. Ruzzonetti, 44, had never even donated blood, but the story so captivated him that two days later he called Riverside Community Hospital to ask how he might do the same thing.

Halfway across the country, in Joliet, Ill., Donald C. Terry Jr. needed a kidney in the worst way. Since receiving a diagnosis of diabetes related renal disease in his mid-40s, he had endured the burning and haunting and dismal tedium of dialysis for nearly a year. With nobody in his family willing or able to give him a kidney, his doctors warned that it might take five years to crawl up the waiting list for an organ from a deceased donor.
Stable marriage problem

- Given n men and n women, where each man ranks all women and each woman ranks all men, find a stable matching.

- **Stable matching**: no pair X and Y (not matched to each other) who prefer each other over their matched partners.
  - Such X & Y: "blocking pair"

- **Perfect matching**
  - Everyone is matched (monogamous)
  - Necessary condition: # men = # women

Is there a stable perfect matching?

- Yes, Gale-Shapley algorithm (1962) [On board]
Gale-Shapley algorithm

- Thm 1.2.1. The algorithm terminates with a stable matching.
- Thm 1.2.2. Men-proposing version is men-optimal [ordering of men doesn’t matter]
- Thm 1.2.3. Men proposing version is the worst for women [each woman gets the worst man subject to the matching being stable]
Applications
beyond kidney exchange

Residency matching

Hospitals interview candidates and rank them

Candidates rank hospitals that interviewed them
How Game Theory Helped Improve New York City’s High School Application Process

By TRACY TULLIS DEC. 5, 2014

Tuesday was the deadline for eighth graders in New York City to submit applications to secure a spot at one of 426 public high schools. After months of school tours and tests, auditions and interviews, 75,000 students have entrusted their choices to a computer program that will arrange their school assignments for the coming year. The weeks of research and deliberation will be reduced to a fraction of a second of mathematical calculation: In just a couple of hours, all the sorting for the Class of 2019 will be finished.

To middle-school students and their parents, the high-school admissions process is a grueling and universally loathed rite of passage. But as awful as it can be, it used to be much worse. In the late 1990s, for instance, tens of thousands of children were shunted off to schools that had nothing going for them, it seemed, beyond empty desks. The process was so byzantine it appeared nothing short of a Nobel Prize-worthy algorithm could fix it.

NYC high school matching

- Around 80K 8-th graders are matched to around 500 high schools
- Each student ranks at most 12 schools
- Schools rank applicants
  - ‘But schools continue to tell parents and students — “with a wink” — that they may be penalized if they don't list their school first.’ (https://www.dnainfo.com/new-york/20161115/kensington/nyc-high-school-admissions-ranking)
- Match by DOE
Content delivery networks (CDN)

Algorithmic Nuggets in Content Delivery

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ABSTRACT
This paper "peeks under the covers" at the subsystems that provide the basic functionality of a leading content delivery network. Based on our experiences in building one of the largest distributed systems in the world, we illustrate how sophisticated algorithmic research has been adapted to balance the load between and within server clusters, manage the caching on servers, select paths through an overlay routing network, and elect leaders in various contexts. In each instance, we first explain the theory underlying the algorithms, then introduce practical considerations not captured by the theoretical models, and finally describe what is implemented in practice. Through these examples, we highlight the role of algorithmic research in the design of complex networked systems. The paper also illustrates the close synergy that exists between research and industry where research ideas cross over into products and product requirements drive future research.

1. INTRODUCTION
The top-three objectives for the designers and operators of a content delivery network (CDN) are high reliability, great end-user experience, and low operating costs. This paper focuses on the first objective.

Matching market
Starter model: Buyers mark goods acceptable or not
Bipartite matching problem

Each link: The room is “acceptable” by the student

Perfect matching

Choice of edges in the bipartite graph so that each node is the endpoint of exactly one of the chosen edges.

Dark edges are the chosen edges—also known as the assignment.
Perfect matching: more examples

A bipartite graph

One perfect matching

Another perfect matching

Constricted set

- A set of nodes $S$ is constricted if its neighbor set $N(S)$ has less number of nodes
  - $|N(S)| < |S|$
- Constricted set $\Rightarrow$ Perfect matching is impossible
- Reverse is also true!
Matching Theorem/Hall's Theorem
Konig (1931), Hall (1935)

- Gives a characterization of perfect matching
- A bipartite graph (with equal numbers of nodes on the left and right) has no perfect matching if and only if it contains a constricted set.

But not all dorm rooms are same...
Model with valuations

- Each student has a valuation for each room
- Find a perfect matching that maximizes the sum of the valuations
  - Social welfare = sum of the valuations
Model with valuations

- Many different perfect matchings:

Room 1
- Alice: 70, 20, 30

Room 2
- Bob: 60, 20, 0

Room 3
- Cindy: 50, 40, 10

Social welfare = 130

Social welfare = 100

Social welfare = 110

How to find a perfect matching that maximizes the social welfare?

Optimal assignment

More general matching markets

Valuations and optimal assignment
Model

- n sellers, each is selling a house
  - $p_i =$ price of seller $i$’s house
- n buyers
  - $v_{ij} =$ buyer $j$’s valuation of seller $i$’s house (or house $i$)
  - $(v_{ij} - p_i)$ is buyer $j$’s payoff if he buys house $i$
- Assumption: buyers are not stupid
  - Maximize their payoffs
  - Maximum payoff must also be $\geq 0$
- Preferred seller graph
  - Bipartite graph between buyers and sellers where every edge encodes a buyer’s maximum payoff ($\geq 0$)

What we want

- A perfect matching in the preferred seller graph
  - Market clearing prices (MCP): The set of prices at which we get a perfect matching
- It would be awesome if the perfect matching is also an optimal assignment!
  - Maximizes social welfare (i.e., sum of the buyers’ valuations in that assignment)
Next

- Show: Any MCP gives an optimal assignment
- Does an MCP always exist?
  - Constructive proof (by an algorithm)
Algorithm for Market clearing price (MCP)

- MCP: prices for which there exists a perfect matching in the preferred seller graph
- Algorithm
  1. Initialize prices to 0
  2. Buyers react by choosing their preferred seller(s)
  3. If resulting graph has a perfect matching then done!
     Otherwise, the neighbors of a constricted set increase price by 1 unit;
     (Normalize the prices—by decreasing all prices by the same amount so that at least one price is 0);
     Go to step 2
- MCP maximizes each buyer's payoff as well as the social welfare

2nd price auction

- Single-item auction is a matching market!

(a) Start of the Auction

(b) End of the Auction