How do senators vote?

- Issues
- Affiliation

- Strategies
- Influence

Voting Behavior
Stat: Ideal Point Models (Davis+, 1970)

Parameters:
- \( p_i \) – *ideal point* of senator \( i \)
- \( a_d \) – *polarity* of bill \( d \)
- \( b_d \) – *popularity* of bill \( d \)

Variable:
- \( x_{id} \) – yea (+1) or nay (-1)

\[
p(x_{i,d} = \text{yea}|p_i, a_d, b_d) = \sigma(p_i a_d + b_d).
\]

---

Linear Influence Game \((\text{LIG})\) (Irfan & Ortiz, 2014)
Strengths & Weaknesses of Models

<table>
<thead>
<tr>
<th>Model:</th>
<th>Strengths:</th>
<th>Weaknesses:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Influence Game (LIG)</td>
<td>Strategic behavior</td>
<td>Bill-specific voting</td>
</tr>
<tr>
<td>Ideal Point</td>
<td>Bill-specific voting</td>
<td>Strategic behavior</td>
</tr>
</tbody>
</table>

2.3.2 Influence Network Models

In contrast to ideal point models, influence network models excel at modeling the social interactions of legislators, while largely ignoring individual preferences. While ideal point models can fail to capture instances where lawmakers deviate from their agenda positions, influence network models are designed to do exactly that. However, the main limitation of influence network models is that they lack any notion of how a lawmaker's preferences on issues influence how they vote. Specifically, the idea that legislators could be more or less likely to vote yea on certain bills. While the threshold parameter does capture a legislator's rigidity towards influence, it does not interact with any bill-specific parameters in a manner similar to how ideal points interact with bill polarities. This leaves a potential loss of information in the model: lawmakers will adjust their voting behavior not just because of the behavior of others, but also in response to the bills themselves. This contrasts the ideal point models, which use the interaction between the polarity and ideal point parameters to characterize legislators voting based on their agenda positions.

3 An Ideal Point Model with Social Interactions

The model that we construct in the following sections uses the different strengths of the other models to produce a more accurate representation of Senate voting behavior. While ideal point models struggle to represent social interactions and influence networks do not account for lawmaker issue preferences, our model captures how senators react to legislation and how they interact with each other. The construction of the our model can be seen as an extension of either an ideal point or an influence network model. In the first case, the our model expands the bill-specific popularity parameter. As mentioned earlier, the popularity abstracts away social interactions into a single number for each bill. Our model replaces this term with an influence network model of social interactions, thereby capturing the same information, but in much more detail. In the second case, parameters for senator ideal points and bill polarities are added to the influence network model. These parameters then interact with each other depending on senators' issue preferences, and cause senators' best responses to change depending on the interaction between their ideal points and a bill's polarity.

Our implemented model more closely resembles the second case: we started with Honorio and

<table>
<thead>
<tr>
<th>Terms</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_i \in X )</td>
<td>Votes of senator ( i )</td>
</tr>
<tr>
<td>( w_{i,-i} \in W )</td>
<td>Incoming influence on senator ( i ) from all other senators (-i)</td>
</tr>
<tr>
<td>( t_i )</td>
<td>Influence threshold of senator ( i )</td>
</tr>
<tr>
<td>( p_i )</td>
<td>Ideal point of senator ( i )</td>
</tr>
<tr>
<td>( a_l )</td>
<td>Polarity of bill ( l )</td>
</tr>
<tr>
<td>( m )</td>
<td>Number of bills</td>
</tr>
<tr>
<td>( \ell_d \in D )</td>
<td>The topics in each bill ( d ) (Sec. (3.2.2))</td>
</tr>
</tbody>
</table>
Ideal Point Model with Social Interactions

- Influence function of Senator $i$ for bill $l$

$$f_i(x_{-i}, l) \equiv \sum_{j \in N_i} w_{ij} x_j - t_i + (p_i \cdot a_l)$$

- Influence function $> 0 \Rightarrow$ B.R. is vote yea
- Influence function $< 0 \Rightarrow$ B.R. is vote nay
- Influence function $= 0 \Rightarrow$ Indifferent

Voting Data

<table>
<thead>
<tr>
<th>Vote</th>
<th>Alexander</th>
<th>Ayotte</th>
<th>Baldwin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Learn Parameters

Data

Model does not immediately predict anything

Must compute equilibria first

NP-Hard (Irfan & Ortiz, 2014)
Implementation Issues: scaling

- Warren (D-MA) -4
- Sanders (I-VT) -4
- Shelby (R-AL) +4
- Cruz (R-TX) +4

Implementation Issues: polarity of unseen bills

New bill

Polarity = ?

Euclidean dist. to existing bills
Selected Model:
\[ \rho = 0.0225 \]
\[ \rho' = 0.004 \]
\[ \approx 1000 \text{ edges} \]
16% validation error

Model Evaluation
Our model vs. LIG model
Criterion: % data that are NE

100% Test Accuracy → Empty Model

Problem: large # of NE

Want

% data as eq.

# eq.
Model Evaluation

- True proportion of equilibria:
  \[ \pi(G) \equiv |\mathcal{NE}(G)|/2^N \]

- Proportion of equilibria in data:
  - \( q = \) fraction of observed data captured as NE
  \[ \frac{q \cdot m}{|\mathcal{NE}(G)|/2^N} \rightarrow \log \left( \frac{q}{|\mathcal{NE}(G)|} \right) \]

Model Evaluation

LIG model:
- \( q = 20\% \)
- \( |\text{NE}(G)| = 287,494 \)

\[ \log_{10} \left( \frac{q}{|\mathcal{NE}(G)|} \right) = \log_{10} \frac{20}{287494} \approx -4.16 \]

Our model:
- \( q = 4.83\% \)
- \( |\text{NE}(G)| = 3,242 \)

\[ \log_{10} \left( \frac{q}{|\mathcal{NE}(G)|} \right) = \log_{10} \frac{4.83}{3242} \approx -2.69 \]

(Our model constrained to ensure similar graph size)
114th U.S. Senate
January 2015 – January 2017

Ideal Point

[Graph showing the Ideal Point for senators]
Most influential senators: group of senators who can enforce a desirable outcome

Most influential senators (aᵢ = 0)
Case Studies
Three bills (Not in training set)

Bill polarities
Case Study: Keystone XL Pipeline

- Passed 62 - 36
- Polarity: 1.426
- LIG Model:
  - 287,400 NE
  - Median correct votes: 50
  - 0.005% of eq. had at least 90
- Our Model:
  - Only one possible NE
  - 91 correct votes

Case Study: Amdt. 777

“To establish a deficit-neutral reserve fund to recognize that climate change is real and caused by human activity and that Congress needs to take action to cut carbon pollution.”

- Failed 49 - 50
- Polarity: -3.705
- LIG Model:
  - 287,400 NE
  - Median correct votes: 50
  - 0.005% of eq. had at least 90
- Our Model:
  - Only one possible NE
  - 92 correct votes
Case Study: Motion to Invoke Cloture

- Passed 84-12
  - Cruz, Paul, Warren, and Sanders voted *yea*
- Polarity: -0.0297
- LIG Model:
  - 287,400 NE
  - Median correct votes: 44
- Our Model:
  - 3,200 NE
  - Median correct votes: 77

**Main Take-Aways**

- Significant improvement in quality of equilibria
- Stronger predictive power than LIG model
- Ability to adjust for bills leads to specific NE
- Improved computation time
Future Work

- More efficient equilibrium computation
- Topic modeling
- Bayesian games